Robust stabilization of uncertain singular systems via output feedback

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Abstract. A series of uncertain singular systems with time-invariant parameter uncertainties is presented, which solves the difficult problem of output feedback robust stabilization control of the system. By using linear matrix inequality (LMI) method, the condition of solvable problem is obtained, and the corresponding output feedback control rate is presented. The obtained theorem not only deepens the robust control theory, but also greatly simplifies the previous methods. Under certain conditions, the robust output feedback control rate can make the closed-loop system regular, causal and asymptotically stable for all admissible uncertain parameters.

Key words. Robust, output, feedback, stabilization, uncertain singular systems.

1. Introduction

Since the concept of Uncertain Singular System was proposed the first time in 1974 by Rosenbrock, the research on Uncertain Singular Systems theory has made greatly progress. In the current documents, Singular System is often called Singular System, Descriptive Variable Systems, Semi-state System, Differential-algebraic Equations. Singular System is the unique name in this article to avoid confusion. Considering the fact that the uncertainty of the actual system always exists under the influence of various factors, the robustness analysis and robust design of the Uncertain Singular System have attracted the scholars' attention in recent years. However, compared with normal systems, the research results of robust control of Uncertain Singular System is few. The existing conclusions are not widely applicable with the shortcomings that existing conclusions cannot deal with time-varying parameter perturbation, the design method is more complex, the design process involves the decomposition of the system matrix, more design steps are needed, etc.In, with the necessary and sufficient conditions for state feedback robust stabilization

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are obtained by using the concept and criteria of uncertain singular quadratic stability. Sometimes, however, the state of the system is often difficult to measure. Even worse, it is impossible to measure in some cases. Therefore, the output feedback stabilization is always an important step in control theory and application research. Therefore, the output feedback robust stabilization of Uncertain Singular System is the point in this paper. Compared with the traditional methods, method applied in this paper not only deepens the robust control theory, but also greatly simplifies the calculation of the matrix and avoids the complicated parameter adjustment, which brings convenience to the practical application[1-5].

2. Problem description

Consider the singular system

$$E\dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

Where $x(t) \in R^n$,$u(t) \in R^m$,$y(t) \in R^p$are respectively state vector, control input and output vector of the system;$E \in R^{n \times n}$and $rankE = r \leq n$;$A, B, C$are known constant matrix with proper order, $\Delta A(t), \Delta B(t)$ are respectively probabilistic actual value matrix functions of state matrix and input matrix, and meet the form as follow:

$$\begin{bmatrix} \Delta A(t) & \Delta B\ (t) \end{bmatrix} = MF(t) \begin{bmatrix} N_a & N_b \end{bmatrix} \tag{3}$$

The matrix $F(t) \in R^{j \times k}$ is unknown real bounded function matrix, and meets

$$F^T(t)F(t) \leq I \tag{4}$$

Here I is $n \times n$ unit matrix,M, N_a, N_b are known matrixes with proper order. Say the uncertain parameters$\Delta A(t), \Delta B(t)$which meet the conditions above are admissible.

Suppose state variable $x(t)$ is completely observable in uncertain singular systems (1) (2), and N_b is full rank. $rankE = r \leq n$, hence it is still general, suppose E has the following form:

$$E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \tag{5}$$

Suppose static output feedback control has the following form:

$$u(t) = Ky(t) \tag{6}$$

Here $K \in R^{m \times p}$ is the undetermined output feedback gain matrix. Then the model of uncertain closed-loop system with probabilistic parameter is

$$E\dot{x}(t) = [A + \Delta A(t) + (B + \Delta B(t))KC]\,x(t) \tag{7}$$

The aim of this paper is to find the feedback gain matrix K, which makes the closed-loop system (7) is regular, without pulse but with robustness to all admissible uncertain $\Delta A(t), \Delta B(t)$.

It is convenient to introduce the following definition for further discussion.

2.1. Definition 1

If uncertain singular system

$$E\dot{x}(t) = (A + \Delta A(t))x(t) \tag{8}$$

is regular, without pulse and stable to all admissible uncertain parameter ΔA, then the uncertain singular system (8) is robust stable.

Hence, it is easy to get the definition below.

2.2. Definition 2

If there is static output feedback control law $u(t) = Ky(t)$ which makes the closed-loop system (7) is robust stable to all admissible uncertain$\Delta A(t), \Delta B(t)$,then the closed-loop system (7) is robust controllable, and $u(t) = Ky(t)$ is one robust output feedback control law of the uncertain singular system (1) (2) .

2.3. Lemma1

If there is constant matrix$P$and positively definite matrix$Q$, makes the uncertain singular system (8) meet $A_0^T(t)P + P^T A_0(t) \leq -Q$to all admissible uncertain parameter $\Delta A(t)$, here $A_0 = A + \Delta A(t)$,P has the following form:
$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$, Among them $P_1 \in R^{r \times r}, P_2 \in R^{(n-r) \times r}, P_3 \in R^{(n-r) \times (n-r)}$and $P_1 > 0$,$P_3$is reversible, then uncertain singular system (8) is robust stable.

Proof: Under the condition of lemma 1, uncertain singular system (8) is uncertain singular quadratic stable, therefore uncertain singular system (8) is robust stable.

QED.

3. Main results

3.1. Theorem 1

There exists static output control law $u(t) = Ky(t)$, which makes closed-loop system (7) robust controllable to all admissible uncertain$\Delta A(t), \Delta B(t)$,that is, the necessary and sufficient condition for that $u(t) = Ky(t)$ is the robust static output feedback control law of uncertain singular system (1) (2) is existing matrix P with the following form: $P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$. Among them, $P_1 \in R^{r \times r}, P_2 \in R^{(n-r) \times r}, P_3 \in$

$R^{(n-r)\times(n-r)}$and $P_1 > 0$,$P_3$is reversible, and$\delta, \lambda > 0$, meet theinequation below:

$$A^T P + P^T A + \delta P^T M M^T P - \lambda^2 P^T B B^T P + \Omega^T \Omega < 0 \qquad (9)$$

Among them, $\Omega = \begin{bmatrix} \delta^{-1/2}(N_a + N_b KC) \\ \lambda B^T P + \lambda^{-1} KC \end{bmatrix}$

It is convenient for proof to introduce the following lemma.

3.1.1. Lemma2 TakeD, E, F are real matrixes with proper order,$\|F(t)\| \leq 1$, then to any $\varepsilon > 0$, there is$DF(t)E + E^T F^T(t)D^T \leq \varepsilon^{-1} D D^T + \varepsilon E^T E$.

3.1.2. Lemma3 To any $x \in R^n$, there is $\max \left\{ \left(x^T M F(t) N x\right)^2 : F^T(t)F(t) \leq I \right\} = \left(x^T M M^T x\right)\left(x^T N^T N x\right)$

3.1.3. Lemma4 If $M, N, P \in R^{n \times n}$are given symmetrical matrixes, meet $M \geq 0, N \geq 0, P < 0$, and $\left(x^T P x\right)^2 - 4\left(x^T M x\right)\left(x^T N x\right) > 0$.when $x \in R^n, x \neq 0$.Then there is constant $\lambda > 0$ makes$P + \lambda M + \lambda^{-1} N < 0$.

3.1.4. theorem 1 **Sufficiency:**
To closed-loop system (7), take

$$A_C(t) = A + \Delta A(t) + (B + \Delta B(t))KC) = A + BKC + MF(t)(N_a + N_b)KC$$

Therefore, from lemma 2, there must be$\delta >, \lambda > 0$, and

$$A_C^T(t)P + P^T A_C(t) = ([A + BKC + MF(t)(N_a + N_b KC)]^T)P + [(N_a + N_b KC)]$$

$$= (A+BKC)^T P + P^T(A+BKC) + [MF(t)(N_a + N_b KC)]^T P + P^T [MF(t)(N_a + N_b KC)]$$

$$\leq (A+BKC)^T P + P^T(A+BKC) + \delta P^T M M^T P + \frac{1}{\delta}(N_a + N_b KC)^T(N_a + N_b KC)$$

$$\leq A^T P + P^T A + \delta P^T M M^T P + C^T K^T B^T P + P^T BKC + \frac{1}{\delta}(N_a + N_b KC)^T + \frac{1}{\lambda^2}C^T K^T KC$$

$$\leq A^T P + P^T A + \delta P^T M M^T P + \frac{1}{\delta}(N_a + N_b KC)^T + (\lambda B^T P + \lambda^{-1}KC)^T(\lambda B^T P + \lambda^{-1}KC)$$

Take $\Omega = \begin{bmatrix} \delta^{-1/2}(N_a + N_b KC) \\ \lambda B^T P + \lambda^{-1} KC \end{bmatrix}$,
Then$A_C^T(t)P + P^T A_C(t) \leq A^T P + P^T A + \delta P^T M M^T P - \lambda^2 P B B^T P + \Omega^T \Omega$.

Hence, from the theorem condition and lemma 1, the closed-loop system (7) is robust controllable, in other words, $u(t) = Ky(t)$ is one robust static output feedback control law of uncertain singular system(1) (2).

Necessity:

If exist static output feedback control law $u(t) = Ky(t)$, makes closed-loop system (7) is robust controllable to all admissible uncertain ΔA, learn from lemma 1 that there are positive definite matrix Q_1 and matrix $P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$, among them

$P_1 \in R^{r \times r}, P_2 \in R^{(n-r) \times r}, P_3 \in R^{(n-r) \times (n-r)}$, and $P_1 > 0, P_3$ is reversible, meet$A_C^T(t)P + P^T A_C(t) \leq -Q_1$,

Take$Z = (A + BKC)^T P + P^T(A + BKC)$,then from the above inequality, to any non-zero$x \in R^n$and all admissible$F(t)$, there is

$x^T Z x < -2x^T P^T \left[MF(t)(N_a + N_b KC) \right] x$, that is

$$x^T Z x < -2 \max \left\{ x^T P^T \left[MF(t)(N_a + N_b KC) \right] x : F^T(t)F(t) \leq I \right\} \leq 0$$

According to Lemma 3

$$(x^T Z x)^2 > 4 \max \left\{ \left[x^T P^T \left[MF(t)(N_a + N_b KC) \right] x \right]^2 : F^T(t)F(t) \leq I \right\}$$

$$= 4 \left[x^T (P^T M M^T P) x \right] \bullet \left[x^T (N_a + N_b KC)^T (N_a + N_b KC) x \right]$$

Further, known from lemma 4, there is constant$\delta > 0$meet

$$Z + \delta P^T M M^T P + \delta^{-1}(N_a + N_b KC)^T (N_a + N_b KC) < 0,$$

That is

$$(A + BKC)^T P + P^T(A + BKC) + \delta P^T M M^T P + \delta^{-1}(N_a + N_b KC)^T (N_a + N_b KC)$$

$$= A^T P + P^T A + \delta P^T M M^T P + \frac{1}{\delta} N_a^T N_a + C^T K^T (P^T B + \frac{1}{\delta} N_a^T N_b)^T + C^T K^T \frac{1}{\delta} N_b^T N_b KC$$

$$+ (P^T B + \frac{1}{\delta} N_a^T N_b) KC < 0$$

Take$\Omega = \begin{bmatrix} \delta^{-1/2}(N_a + N_b KC) \\ \lambda B^T P + \lambda^{-1} KC \end{bmatrix}$, according to the inequation above,

$$A^T P + P^T A + \delta P^T M M^T P - \lambda^2 P^T B B^T P + \Omega^T \Omega < 0.$$

Consider the example of$P$, the necessity is proved.

QED.

3.2. Theorem 2

Suppose $C \in R^{n \times n}$is reversible matrix,take static output feedback control law $u(t) = -\left[(N_b^T N_b)^{-1}(B^T S^{-T} + N_b^T N_a)C^{-1}\right] y(t)$, then closed-loop system(??)is robust controllable to all admissible uncertain $\Delta A, \Delta B$, that is, the necessary and sufficient condition for that$u(t) = Ky(t)$is the robust static output feedback control law of uncertain singular system (1) (2) is existing matrix S which with the following form:$S = \begin{bmatrix} S_1 & S_2 \\ 0 & S_3 \end{bmatrix}$,

Among them, $S_1 \in R^{r \times r}, S_2 \in R^{(n-r) \times r}, S_3 \in R^{(n-r) \times (n-r)}andS_1 > 0, S_3$ is reversible, meet the linearmatrix inequation below:

$$\begin{bmatrix} SA_1^T + A_1 S^T - B(N_b^T N_b)^{-1}B^T & SN_a^T U^T & M \\ UN_a S^T & -I_{k \times k} & 0 \\ M^T & 0 & -I_{j \times j} \end{bmatrix} < 0 \qquad (10)$$

Among them,$A_1 = A - N_b(N_b^T N_b)^{-1}N_b^T N_a$, $U^T U = I - N_b(N_b^T N_b)^{-1}N_b^T N_a$, $U \in R^{r \times r}$.

Proof:It is easy to know from the proof of theorem 1, closed-loop system (7) robust controllable to all admissible uncertain$\Delta A(t), \Delta B(t)$, that is, $u(t) = Ky(t)$ is the robust static output feedback control law of uncertain singular system (1) (2) , if and only if there is matrix$P$and positive numberδ,Phas the following form:$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$.

Among them $P_1 \in R^{r \times r}, P_2 \in R^{(n-r) \times r}, P_3 \in R^{(n-r) \times (n-r)}$ and $P_1 = P_1^T > 0, P_3$ is reversible, meet the following inequation,

$$A_C^T(t)P + P^T A_C(t)$$

$$\leq A^T P + P^T A + \delta P^T M M^T P + \frac{1}{\delta} N_a^T + K^T (P^T B + \frac{1}{\delta} N_a^T N_b)^T + (P^T B + \frac{1}{\delta} N_a^T N_b)KC < 0$$

Take$u(t) = -\left[(N_b^T N_b)^{-1}(B^T S^{-T} + N_b^T N_a)C^{-1}\right] y(t)$and $S = (\delta P)^{-T}$, result from above that

$$SA^T + AS^T + MM^T + SN_a^T N_a S^T - SN_a^T N_b(N_b^T N_b)^{-1}B^T$$

$$-B(N_b^T N_b)^{-1}B^T - B(N_b^T N_b)^{-1}N_b^T N_a S^T - SN_a^T N_b(N_b^T N_b)^{-1}N_b^T N_a S^T < 0 \quad (11)$$

Consider$N_b$is full rank, easy to see $I - N_b(N_b^T N_b)^{-1}N_b^T \geq 0$, hence exist $U \in R^{r \times r}$, make

$$U^T U = I - N_b(N_b^T N_b)^{-1}N_b^T N_a.$$

Therefore according to (11), $SA_1^T + A_1 S^T + MM^T + SN_a^T U^T U N_a S^T < 0$

From the expression above and schur lemma can learn that expression (**??**)is true. Consider $S = (\delta P)^{-T}$, obviously S meets the form and request in theorem.

QED.

4. Simulation example

First offer one feasibility algorithm for the above question.

Step 1: use LMI software package to find matrix P, S and $\delta, \lambda > 0$, Take $P > 0$, $S = (\delta P)^{-T}$;

Step 2: use Matlab to find $A1$, U;

Step 3: Find $K = -\left[(N_b^T N_b)^{-1}(B^T S^{-T} + N_b^T N_a)C^{-1}\right]$;

Step 4: Check $A^T P + P^T A + \delta P^T M M^T P - \lambda^2 P^T B B^T P + \Omega^T \Omega < 0$;

If it is true, K is found; or go back to step 1.

Example Consider uncertain singular system (1) (2) , their parameter matrixes are respectively

$$E = \begin{bmatrix} 1 & 0 & 0.5 \\ 2 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 2.4 & 0.2 & 1.2 \\ 4 & 1.5 & 2 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}, M = \begin{bmatrix} 0.1 \\ 0.2 \\ -0.1 \end{bmatrix},$$

$$N_a = \begin{bmatrix} 0.35 & 0.4 & 0.7 \end{bmatrix}, N_b = \begin{bmatrix} 0.1 & 0.2 & 0 \end{bmatrix}, C = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},$$

One output feedback control law obtained from the above algorithm is

$$u(t) = Kx(t) = \begin{bmatrix} -3.1520 & -2.3141 & -3.5567 \\ 4.7984 & 4.9107 & 9.1222 \\ -6.3742 & -5.5566 & -11.7887 \end{bmatrix} x(t)$$

5. Conclusions

This paper provided the necessary and sufficient condition of uncertain singular system output feedback robust control problem with the help of linear matrix inequation(LMI) and Schur nature, and according to this condition designed the expected output feedback control law to realize the robust control of this system. The example in practical engineering further proved this theorem.

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